Date: 22 Feb 2021

VIRTUAL COACHING CLASSES
ORGANISED BY BOS (ACADEMIC), ICAI

FOUNDATION LEVEL
PAPER 3: BUSINESS MATHEMATICS, LOGICAL REASONING & STATISTICS

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PERMUTATION

- 1. X can go from his office to his residence by 3 routes. In how many ways can he go to his residence via one route and come to office by another route ?
- R1, R2, R3
- 3*2 = 6 ways

- 2. there are 6 boxes & 3 balls. In how many ways these 3 balls may be put into these 6 boxes, not more than 1 ball in each box
- Ball 1: 6 ways
- Ball 2 : 5 ways
- Ball 3: 4 ways
- Total= 6*5*4= 120 = 6P3

Selection of 2 units out of 4 units

- In how many ways 2 alphabets may be selected out of A, B, C D ?
- AB, AC AD, BC, BD, CD
- BA, CA, DA , CB, DB, DC
- Please note : AB and BA are **2 separate selections**

Permutations vs. Combinations

- Both are ways to count the possibilities
- Arrangements
- The difference between them is whether order matters or not

Permutations

- A permutation is an ordered arrangement of the elements of some set *S*
  - *Let S = {a, b, c}*
  - *c, b, a is a permutation of S*
  - *b, c, a is a different permutation of S*
- The notation for the number of *r*-permutations: $P(n,r)$

Permutations - FUNDAMENTAL PRINCIPLES OF COUNTING

- **Multiplication Rule:** If certain thing may be done in 'm' different ways and **when it has been done**, a second thing can be done in 'n ' different ways then total number of ways of **doing both things simultaneously**
- **= m × n.**
- Eg. if one can go to school by 5 different buses and then come back by 4 different buses then total number of ways of going to and coming back from school = 5 × 4 = 20.
- **Addition Rule :** It there are two **alternative jobs** which can be done in 'm' ways and in 'n' ways respectively **then either of two jobs can be done in (m + n) ways.**
- Eg. if one wants to go school by bus where there are 5 buses or to by auto where there are 4 autos, then total number of ways of going school = 5 + 4 = 9.

Permutations

The notation P(n,r) represents the number of permutations (arrangements) of n objects taken r at a time when r is less than or equal to n. In a permutation, the **order is important.**

In our example, we have P(3,2) which represents the number of permutations of 3 objects taken 2 at a time.

In our case, P(3,2) = 6 = (3)(2)

In general, P(n,r) = n(n-1)(n-2)(n-3)…(n-r+1)

- **Number of Permutations when r objects are chosen out of n different objects. (Denoted by**
- nPr or nPr or P(n, r)) :
- nPr using factorial notation.
- nPr = n. (n – 1) (n – 2) ….. (n – r + 1)
- <u> (n - r) (n - r - 1) 2.1</u>
-

  = n (n – 1) (n – 2) ….. (n – r + 1) ×
-

  1.2 ...(n - r - 1) (n - r)
-

  = n!/(n – r)!

THE FACTORIAL

- **Definition:** The factorial n, written as n! or n , represents the product of all integers from 1 to n both inclusive.
- To make the notation meaningful, when n = o, we define o! or o = 1.
- Thus, n! = n (n – 1) (n – 2) 3.2.1

Definition of n factorial (!)

- $n! = n(n-1)(n-2)(n-3)\ldots 1$
- For example, $5! = 5(4)(3)(2)(1)=120$
- $0! = 1$ by definition.
- **8 !   = 8(7)(6)(5)(4)(3)(2)(1)=8! =40,320**.

More examples

Use the definition **P(n,r) = n(n-1)(n-2)(n-3)…(n-r+1)**

- **Find P(5,3)**
- Here, n = 5 and r = 3 so we have P(5,3) = (5)(5-1)5-3+1) =
- 5(4)3 = 60. This means there are 60 arrangements of 5 items taken 3 at a time.
- **Application: How many ways can 5 people sit on a park bench if the bench can only seat 3 people?**
- Solution: Think of the bench as three slots ___ ___ ___ .
- There are five people that can sit in the first slot, leaving four remaining people to sit in the second position and finally 3 people eligible for the third slot. Thus, there are 5(4)(3)=60 ways the people can sit. The answer could have been found using the permutations formula: P(5,3) = 60, since we are finding the number of ways of arranging 5 objects taken 3 at a time.

$P(n,n) = n(n-1)(n-2)\ldots1$

- **Find P(5,5)** , the number of arrangements of 5 objects taken 5 at a time.
- Answer: $P(5,5) = 5(5-1)\ldots(5-5+1) = 5(4)(3)(2)(1)=120$.
- **Application: A bookshelf has space for exactly 5 books. How many different ways can 5 books be arranged on this bookshelf?**
- ___ ___ ___ ___ ___ Think of 5 slots, again. There are five choices for the first slot, 4 for the second and so on until there is only 1 choice for the final slot. The answer is 5(4)(3)(2)(1)

  which is the same as $P(5,5) = 120$.

PERMUTATION CASE

- **Ex 5 – pg 5.3 of study material**
- Suppose the group consists of Mr. Suresh, Mr. Ramesh and Mr. Mahesh. Then how many films does the photographer need? He has to arrange three persons amongst three places with due regard to order.
- Then the various possibilities are (Suresh, Mahesh, Ramesh), (Suresh, Ramesh, Mahesh), (Ramesh, Suresh, Mahesh), (Ramesh, Mahesh, Suresh), (Mahesh, Ramesh, Suresh) and (Mahesh, Suresh, Ramesh).
- Thus there are six possibilities. Therefore he needs six films. Each one of these possibilities is called a permutation of three persons taken at a time

Alternative	Place 1	Place2	Place 3
1	Suresh……….	Mahesh……….	Ramesh
2	Suresh……….	Ramesh……….	Mahesh
3	Ramesh………	Suresh…………	Mahesh
4	Ramesh………	Mahesh……….	Suresh
5	Mahesh………	Ramesh……….	Suresh
6	Mahesh………	Suresh………….	Ramesh

Examples 6 & 7 – work out (Example 1, pg 5.5)

- Evaluate each of 5P3, 10P2, 11P5.
- **Solution:** $5P3 = 5 \times 4 \times (5-3+1) = 5 \times 4 \times 3 = 60$,
- $10P2 = 10 \times \ldots \times (10-2+1) = 10 \times 9 = 90$,
- $11P5 = 11! / (11 - 5)! = 11 \times 10 \times 9 \times 8 \times 7 \times 6! / 6! = 11 \times 10 \times 9 \times 8 \times 7 = 55440$.

- Ex 8: (Example 2, pg 5.5)
- How many three letters words can be formed using the letters of the words
- (a) SQUARE and (b) HEXAGON?
- (Any arrangement of letters is called a word even though it may or may not have any meaning or pronunciation).
- **Solution:**
- Since the word 'SQUARE' consists of 6 different letters, the number of permutations of choosing 3 letters out of six equals 6P3 = 6 × 5 × 4 = 120.
- Since the word 'HEXAGON' contains 7 different letters, the number of permutations is
- 7P3 = 7 × 6 × 5 = 210.

Ex 9 - example 3, pg5.5

- In how many different ways can five persons stand in a line for a group photograph?
- **Solution:** Here we know that the order is important. Hence, this is the number of permutations of five things taken all at a time. Therefore, this equals
- $5P5 = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ ways.

Ex ample 4, 5 : Pg 5.5.

- **Example 10 :** First, second and third prizes are to be awarded at an engineering fair in which 13 exhibits have been entered. In how many different ways can the prizes be awarded?
- **Solution:** Here again, order of selection is important and repetitions are not meaningful as no exhibit can receive more than one prize. Hence , the answer is the number of permutations of 13 things taken three at a time. Therefore, we find 13P3 = 13!/10! = 13×12×11 = 1,716 ways.
- **Example 11 :** In how many different ways can 3 students be associated with 4 chartered accountants, assuming that each chartered accountant can take at most one student?
- **Solution:** This equals the number of permutations of choosing 3 persons out of 4. Hence , the answer is 4P3 = 4×3×2 = 24.

Ex 13) How many ways can you order a white, chocolate, or yellow cake, with chocolate or vanilla icing, and 20 possible designs on top?

First think about what kind of problem this is. Are there different categories or not? Does the order matter?

This uses the counting principle because there are 3 different categories involved: the type of cake, type of icing, and the type of design.

$3 \bullet 2 \bullet 20 = 120$ possible outcomes

Ex 15 : How many ways can 7 students finish a race in 1st , 2nd, and 3rd place?

First decide if the order matters or not. Then calculate.

1) It is a permutation because you would rather win the race than finish in 2nd or 3rd place.

$${}_{7}P_{3}$$

$\frac{7!}{(7-3)!}$ we know the $(7-3)!$ Cancels 4 and below so

$$\frac{7 \bullet 6 \bullet 5}{1}$$

$\frac{210}{1}$ = 210 possible outcomes

- Ex 16- Find n if nP3 = 60.
- **Solution:**
- n P = $\frac{n!}{(n-3)!}$ = 60 (given)
- i.e., n (n–1) (n–2) = 60 = 5 × 4 × 3
- Therefore, n = 5.

- Pg 5.8 Ex A, No 14 & 15

- Ex 19 - The number of ways the letters of the word `COMPUTER' can be rearranged is
- a) 40,320 b) 40,319 c) 40,318 d) none of these
- Ex 20 - The number of arrangements of the letters in the word `FAILURE', so that **vowels are always coming together is**
  - *576 b) 575 c) 570 d) none of these*

CIRCULAR PERMUTATIONS – Ex 1, pg 5.9

- There are nPn/ n ways in which all the *n* things can be arranged in a circle.
- This equals $(n-1)!$.
- In how many ways can 4 persons sit at a round table for a group discussions?
- **Solution:** The answer can be get from the formula for circular permutations. The answer is $(4-1)! = 3! = 6$ ways.

PERMUTATION WITH RESTRICTIONS

- **Theorem 1. Number of permutations of n distinct objects taken r at a time when a particular object is not taken in any arrangement is $^{n-1}P_r$.**
- **Proof :** Since a particular object is always to be excluded, we have to place n – 1 objects at r places. Clearly this can be done in $^{n-1}P_r$. ways.

Theorem 2

Theorem 2. Number of permutations of r objects out of n distinct objects when a particular object is always included in any arrangement is r. ${}^{n-1}p_{r-1}$

Proof : If the particular object is placed at first place, remaining r – 1 places can be filled from n – 1 distinct objects in ${}^{n-1}p_{r-1}$ ways. Similarly, by placing the particular object in 2nd, 3rd, ….., r^{th} place, we find that in each case the number of permutations is ${}^{n-1}p_{r-1}$. This the total number of arrangements in which a particular object always occurs is r. ${}^{n-1}p_{r-1}$

- **Ex 1, pg 5.10**
- **Example 24:** How many arrangements can be made out of the letters of the word `DRAUGHT', the vowels never beings separated?
- **Solution:** The word `DRAUGHT' consists of 7 letters of which 5 are consonants and two are vowels. In the arrangement we are to take all the 7 letters but the restriction is that the two vowels should not be separated.
- We can view the two vowels as one letter. The two vowels A and U in this one letter can be arranged in 2! = 2 ways. (i) AU or (ii) UA. Further, we can arrange the six letters : 5 consonants and one letter compound letter consisting of two vowels. The total number of ways of arranging them is 6P6 = 6! = 720 ways.
- Hence, by the fundamental principle, the total number of arrangements of the letters of the word DRAUGHT, the vowels never being separated = 2 × 720 = 1440 ways

Example 25 : Pg 5.11, Ex 3

- There are 6 books on Economics, 3 on Mathematics and 2 on Accountancy. In how many ways can these be placed on a shelf if the books on the same subject are to be together?
- **Solution:** Consider one such arrangement. 6 Economics books can be arranged among themselves in 6! Ways, 3 Mathematics books can be arranged in 3! Ways and the 2 books on Accountancy can be arranged in 2! ways. Consider the books on each subject as one unit. Now there are three units. These 3 units can be arranged in 3! Ways.
- Total number of arrangements = $3! \times 6! \times 3! \times 2!$
- = 51,840.

- **Ex 26** : How many four digit numbers can be formed out of the digits 1,2,3,5,7,8,9, if no digit is repeated in any number? How many of these will be greater than 3000?
- **Solution:** We are given 7 different digits and a four-digit number is to be formed using any 4 of these digits. This is same as the permutations of 7 different things taken 4 at a time.
- Hence, the number of four-digit numbers that can be formed = 7P4 = 7 × 6 × 5 × 4 × = 840 ways.
- Next, there is the restriction that the four-digit numbers so formed must be greater than 3,000. Thus, it will be so if the first digit-that in the thousand's position, is one of the five digits 3, 5, 7, 8, 9. Hence, the first digit can be chosen in 5 different ways; when this is done, the rest of the 3 digits are to be chosen from the rest of the 6 digits without any restriction and this can be done in 6P3 ways.
- Hence, by the Fundamental principle, we have the number of four-digit numbers greater than 3,000 that can be formed by taking 4 digits from the given 7 digits = 5 × 6P3 = 5 × 6 × 5 × 4 = 5 × 120 = 600.

- **Ex 27** – pg 5.11, Ex 7
- There are 6 students of whom 2 are Indians, 2 Americans, and the remaining 2 are Russians. They have to stand in a row for a photograph so that the two Indians are together, the two Americans are together and so also the two Russians. Find the number of ways in which they can do so.
- **Solution:** The two Indians can stand together in $^2P_2 = 2! = 2$ ways. So is the case with the two Americans and the two Russians.
- Now these 3 groups of 2 each can stand in a row in $^3P_3 = 3 \times 2 = 6$ ways. Hence by the generalized fundamental principle, the total number of ways in which they can stand for a photograph under given conditions is
- $6 \times 2 \times 2 \times 2 = 48$

- Ex 28 - Six boys and five girls are to be seated for a photograph in a row such that no two girls sit together and no two boys sit together. Find the number of ways in which this can be done.
- **Solution:** Suppose that we have 11 chairs in a row and we want the 6 boys and 5 girls to be seated such that no two girls and no two boys are together. If we number the chairs from left to right, the arrangement will be possible if and only **if boys occupy the odd places and girls occupy the even places in the row. The six odd places from 1 to 11 may filled in by 6 boys in 6P6 ways.**
- Similarly, the five even places from 2 to 10 may be filled in by 5 girls in 5P5 ways.
- Hence, by the fundamental principle, the total number of required arrangements = 6P6 × 5P5 = 6! × 5! = 720 × 120 = 86,400.

Recap

1. P &C
2. Factorial
3. Circular permutations
4. Difference of P& C
5. Problem solution

THANK YOU